Lesson-by-Lesson Review

Intervention If the given examples are not sufficient to review the topics covered by the questions, remind students that the page references tell them where to review that topic in their textbook.

Two-Day Option Have students complete the Lesson-by-Lesson Review on pp. 356–358. Then you can use McGraw-Hill eAssessment to customize another review worksheet that practices all the objectives of this chapter or only the objectives on which your students need more help.

Additional Answers

11. $\sec \theta = \sqrt{10}$, $\cos \theta = \frac{1}{\sqrt{10}}$ or $\frac{\sqrt{10}}{10}$

12. $\cot \theta = -\frac{1}{2\sqrt{6}}$ or $-\frac{\sqrt{6}}{12}$, $\sin \theta = \frac{2\sqrt{6}}{5}$

13. $\csc \theta = -\frac{5}{4}$, $\tan \theta = -\frac{4}{3}$

14. $\cot \theta = \frac{7}{2}$, $\cos \theta = \frac{7}{\sqrt{53}}$ or $\frac{7\sqrt{53}}{53}$

15. $\sec \theta = \frac{5}{2\sqrt{5}}$ or $\frac{\sqrt{5}}{2}$, $\sin \theta = -\frac{1}{\sqrt{5}}$ or $-\frac{\sqrt{5}}{5}$

16. $\cos \theta = -\frac{3}{\sqrt{73}}$ or $-\frac{3\sqrt{73}}{73}$, $\sin \theta = -\frac{8}{\sqrt{73}}$ or $-\frac{8\sqrt{73}}{73}$

Chapter 5 Study Guide and Review *Continued*

Lesson-by-Lesson Review

5-1 Trigonometric Identities (pp. 312–319)

Find the value of each expression using the given information. **11–16. See margin.**

11. $\sec \theta$ and $\cos \theta$; $\tan \theta = 3$, $\cos \theta > 0$
12. $\cot \theta$ and $\sin \theta$; $\cos \theta = -\frac{1}{5}$, $\tan \theta < 0$
13. $\csc \theta$ and $\tan \theta$; $\cos \theta = \frac{3}{5}$, $\sin \theta < 0$
14. $\cot \theta$ and $\cos \theta$; $\tan \theta = \frac{2}{7}$, $\csc \theta > 0$
15. $\sec \theta$ and $\sin \theta$; $\cot \theta = -2$, $\csc \theta < 0$
16. $\cos \theta$ and $\sin \theta$; $\cot \theta = \frac{3}{8}$, $\sec \theta < 0$

Simplify each expression.

17. $\sin^2(-x) + \cos^2(-x)$ **1**
18. $\sin^2 x + \cos^2 x + \cot^2 x$ **$\csc^2 x$**
19. $\frac{\sec^2 x - \tan^2 x}{\cos(-x)}$ **$\sec x$**
20. $\frac{\sec^2 x}{\tan^2 x + 1}$ **1**
21. $\frac{1}{1 - \sin x}$ **$\sec^2 x + \tan x \sec x$**
22. $\frac{\cos x}{1 + \sec x}$ **$\cot^2 x - \cot^2 x \cos x$**

Example 1

If $\sec \theta = -3$ and $\sin \theta > 0$, find $\sin \theta$.

Since $\sin \theta > 0$ and $\sec \theta < 0$, θ must be in Quadrant II. To find sin, first find $\cos \theta$ using the Reciprocal Identity for $\sec \theta$ and $\cos \theta$.

$\cos \theta = \frac{1}{\sec \theta}$ — Reciprocal Identity

$= -\frac{1}{3}$ — $\sec \theta = -3$

Now you can use the Pythagorean identity that includes $\sin \theta$ and $\cos \theta$ to find $\sin \theta$.

$\sin^2 \theta + \cos^2 \theta = 1$ — Pythagorean Identity

$\sin^2 \theta + \left(-\frac{1}{3}\right)^2 = 1$ — $\cos \theta = -\frac{1}{3}$

$\sin^2 \theta + \frac{1}{9} = 1$ — Multiply.

$\sin^2 \theta = \frac{8}{9}$ — Subtract.

$\sin \theta = \frac{\sqrt{8}}{3}$ or $\frac{2\sqrt{2}}{3}$ — Simplify.

5-2 Verifying Trigonometric Identities (pp. 320–326)

Verify each identity. **23–32. See Chapter 5 Answer Appendix.**

23. $\frac{\sin \theta}{1 - \cos \theta} + \frac{\sin \theta}{1 + \cos \theta} = 2 \csc \theta$
24. $\frac{\cos \theta}{\sec \theta} + \frac{\sin \theta}{\csc \theta} = 1$
25. $\frac{\cot \theta}{1 + \csc \theta} + \frac{1 + \csc \theta}{\cot \theta} = 2 \sec \theta$
26. $\frac{\cos \theta}{1 - \sin \theta} = \frac{1 + \sin \theta}{\cos \theta}$
27. $\frac{\cot^2 \theta}{1 + \csc \theta} = \csc \theta - 1$
28. $\frac{\sec \theta}{\tan \theta} + \frac{\csc \theta}{\cot \theta} = \sec \theta + \csc \theta$
29. $\frac{\sec \theta + \csc \theta}{1 + \tan \theta} = \csc \theta$
30. $\cot \theta \csc \theta + \sec \theta = \csc^2 \theta \sec \theta$
31. $\frac{\sin \theta}{\sin \theta + \cos \theta} = \frac{\tan \theta}{1 + \tan \theta}$
32. $\cos^4 \theta - \sin^4 \theta = \frac{1 - \tan^2 \theta}{\sec^2 \theta}$

Example 2

Verify that $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta$.

The left-hand side of this identity is more complicated, so start with that expression.

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta(1 + \cos \theta)}$$

$$= \frac{\sin^2 \theta + 1 + 2\cos \theta + \cos^2 \theta}{\sin \theta(1 + \cos \theta)}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta + 1 + 2\cos \theta}{\sin \theta(1 + \cos \theta)}$$

$$= \frac{1 + 1 + 2\cos \theta}{\sin \theta(1 + \cos \theta)}$$

$$= \frac{2 + 2\cos \theta}{\sin \theta(1 + \cos \theta)}$$

$$= \frac{2(1 + \cos \theta)}{\sin \theta(1 + \cos \theta)}$$

$$= \frac{2}{\sin \theta}$$

$$= 2 \csc \theta$$

5-3 Solving Trigonometric Equations (pp. 327–333)

Find all solutions of each equation on the interval $[0, 2\pi]$.

33. $2 \sin x = \sqrt{2}$ $\frac{\pi}{4}, \frac{3\pi}{4}$

34. $4 \cos^2 x = 3$ 34. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

35. $\tan^2 x - 3 = 0$

36. $9 + \cot^2 x = 12$

37. $2 \sin^2 x = \sin x$ $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

38. $3 \cos x + 3 = \sin^2 x$ π

35. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Solve each equation for all values of x.

39–44. n is an integer.

39. $\sin^2 x - \sin x = 0$ $n\pi, \frac{\pi}{2} + 2n\pi$

40. $\tan^2 x = \tan x$ $n\pi, \frac{\pi}{4} + n\pi$

41. $3 \cos x = \cos x - 1$ $\frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

42. $\sin^2 x = \sin x + 2$ $\frac{3\pi}{2} + 2n\pi$

43. $\sin^2 x = 1 - \cos x$ $2n\pi, \frac{\pi}{2} + n\pi$

44. $\sin x = \cos x + 1$ $\frac{\pi}{2} + 2n\pi, \pi + 2n\pi$

36. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Example 3

Solve the equation $\sin \theta = 1 - \cos \theta$ for all values of θ.

$\sin \theta = 1 - \cos \theta$	Original equation
$\sin^2 \theta = (1 - \cos \theta)^2$	Square each side.
$\sin^2 \theta = 1 - 2 \cos \theta + \cos^2 \theta$	Expand.
$1 - \cos^2 \theta = 1 - 2 \cos \theta + \cos^2 \theta$	Pythagorean Identity
$0 = 2 \cos^2 \theta - 2 \cos \theta$	Subtract.
$0 = 2 \cos \theta(\cos \theta - 1)$	Factor.

Solve for x on $[0, 2\pi]$.

$\cos \theta = 0$ or $\cos \theta = 1$

$\theta = \cos^{-1} 0$ $\qquad$ $\theta = \cos^{-1} 1$

$\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ $\qquad$ $\theta = 0$

A check shows that $\frac{3\pi}{2}$ is an extraneous solution. So the solutions are $\theta = \frac{\pi}{2} + 2n\pi$ or $\theta = 0 + 2n\pi$.

5-4 Sum and Difference Identities (pp. 336–343)

Find the exact value of each trigonometric expression.

45–50. See margin.

45. $\cos 15°$

46. $\sin 345°$

47. $\tan \frac{13\pi}{12}$

48. $\sin \frac{7\pi}{12}$

49. $\cos -\frac{11\pi}{12}$

50. $\tan \frac{5\pi}{12}$

Simplify each expression.

51. $\dfrac{\tan \frac{\pi}{9} + \tan \frac{8\pi}{9}}{1 - \tan \frac{\pi}{9} \tan \frac{8\pi}{9}}$ 0

52. $\cos 24° \cos 36° - \sin 24° \sin 36°$ $\frac{1}{2}$

53. $\sin 95° \cos 50° - \cos 95° \sin 50°$ $\frac{\sqrt{2}}{2}$

54. $\cos \frac{2\pi}{9} \cos \frac{\pi}{18} + \sin \frac{2\pi}{9} \sin \frac{\pi}{18}$ $\frac{\sqrt{3}}{2}$

Verify each identity. 55–58. See margin.

55. $\cos (\theta + 30°) - \sin (\theta + 60°) = -\sin \theta$

56. $\cos \left(\theta + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} (\cos \theta - \sin \theta)$

57. $\cos \left(\theta - \frac{\pi}{3}\right) + \cos \left(\theta + \frac{\pi}{3}\right) = \cos \theta$

58. $\tan \left(\theta + \frac{3\pi}{4}\right) = \frac{\tan \theta - 1}{\tan \theta + 1}$

Example 4

Find the exact value of $\tan \frac{23\pi}{12}$.

$\tan \frac{23\pi}{12} = \tan \left(\frac{5\pi}{4} + \frac{2\pi}{3}\right)$	$\frac{23\pi}{12} = \frac{5\pi}{4} + \frac{2\pi}{3}$
$= \dfrac{\tan \frac{5\pi}{4} + \tan \frac{2\pi}{3}}{1 - \tan \frac{5\pi}{4} \tan \frac{2\pi}{3}}$	Sum Identity
$= \dfrac{1 - \sqrt{3}}{1 - (-\sqrt{3})}$	Evaluate for tangent.
$= \dfrac{1 - \sqrt{3}}{1 + \sqrt{3}}$	Simplify.
$= \dfrac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \dfrac{1 - \sqrt{3}}{1 - \sqrt{3}}$	Rationalize the denominator.
$= \dfrac{4 - 2\sqrt{3}}{1 - 3}$	Multiply.
$= \dfrac{4 - 2\sqrt{3}}{-2}$ or $-2 + \sqrt{3}$	Simplify.

Additional Answers

45. $\frac{\sqrt{6} + \sqrt{2}}{4}$

46. $\frac{\sqrt{2} - \sqrt{6}}{4}$

47. $2 - \sqrt{3}$

48. $\frac{\sqrt{6} + \sqrt{2}}{4}$

49. $\frac{-\sqrt{6} - \sqrt{2}}{4}$

50. $2 + \sqrt{3}$

55. $\cos (\theta + 30°) - \sin (\theta + 60°)$
$= \cos \theta \cos 30° - \sin \theta \sin 30° - (\sin \theta \cos 60° + \cos \theta \sin 60°)$
$= \frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta - \frac{1}{2} \sin \theta - \frac{\sqrt{3}}{2} \cos \theta$
$= -\sin \theta$

56. $\cos \left(\theta + \frac{\pi}{4}\right)$
$= \cos \theta \cos \frac{\pi}{4} - \sin \theta \sin \frac{\pi}{4}$
$= \frac{\sqrt{2}}{2} \cos \theta - \frac{\sqrt{2}}{2} \sin \theta$
$= \frac{\sqrt{2}}{2} (\cos \theta - \sin \theta)$

57. $\cos \left(\theta - \frac{\pi}{3}\right) + \cos \left(\theta + \frac{\pi}{3}\right)$
$= \cos \theta \cos \frac{\pi}{3} + \sin \theta \sin \frac{\pi}{3} + \cos \theta \cos \frac{\pi}{3} - \sin \theta \sin \frac{\pi}{3}$
$= \frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$
$= \cos \theta$

58. $\tan \left(\theta + \frac{3\pi}{4}\right)$
$= \dfrac{\tan \theta + \tan \frac{3\pi}{4}}{1 - \tan \theta \tan \frac{3\pi}{4}}$
$= \dfrac{\tan \theta - 1}{1 - \tan \theta \cdot (-1)}$
$= \dfrac{\tan \theta - 1}{\tan \theta + 1}$

Anticipation Guide

Have students complete the Chapter 5 Anticipation Guide and discuss how their responses have changed now that they have completed the chapter.

Before the Test

Have students complete pp. 79 and 80 of the Study Notebook to review topics and skills presented in the chapter.

Additional Answers

59. $\frac{4\sqrt{2}}{9}, -\frac{7}{9}, -\frac{4\sqrt{2}}{7}$

60. $\frac{4}{5}, -\frac{3}{5}, -\frac{4}{3}$

61. $-\frac{24}{25}, -\frac{7}{25}, \frac{24}{7}$

62. $-\frac{120}{169}, -\frac{119}{169}, \frac{120}{119}$

71. $$\frac{\sin\alpha}{1-\cos\alpha} = \frac{\sin\alpha}{1-\cos\alpha}\cdot\frac{1+\cos\alpha}{1+\cos\alpha} = \frac{\sin\alpha(1+\cos\alpha)}{1-\cos^2\alpha} = \frac{\sin\alpha(1+\cos\alpha)}{\sin^2\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$$

74a. $$d = \frac{\ell}{\frac{1}{\tan\alpha}+\frac{1}{\tan\beta}}$$

74b. $$\frac{\ell}{\frac{1}{\tan\alpha}+\frac{1}{\tan\beta}} = \frac{\ell}{\frac{\cos\alpha}{\sin\alpha}+\frac{\cos\beta}{\sin\beta}} = \frac{\ell}{\frac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}+\frac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta}} = \frac{\ell}{\frac{\cos\alpha\sin\beta+\sin\alpha\cos\beta}{\sin\alpha\sin\beta}} = \frac{\ell\sin\alpha\sin\beta}{\cos\alpha\sin\beta+\sin\alpha\cos\beta}$$

74c. $$\frac{\ell\sin\alpha\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta} = \frac{\ell\sin\alpha\sin\beta}{\sin(\alpha+\beta)}$$

74d. $$\frac{\ell\sin\alpha\sin\beta}{\sin(\alpha+\beta)} = \frac{\ell\sin\alpha\sin\alpha}{\sin(\alpha+\alpha)} = \frac{\ell\sin^2\alpha}{\sin 2\alpha} = \frac{\ell\sin^2\alpha}{2\sin\alpha\cos\alpha} = \frac{\ell\sin\alpha}{2\cos\alpha} = \frac{\ell}{2}\tan\alpha$$

5 Study Guide and Review *Continued*

Lesson-by-Lesson Review

Multiple-Angle and Product-Sum Identities (pp. 346–354)

Find the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ for the given value and interval. **59–62. See margin.**

59. $\cos\theta = \frac{1}{3}, (0°, 90°)$

60. $\tan\theta = 2, (180°, 270°)$

61. $\sin\theta = \frac{4}{5}, \left(\frac{\pi}{2}, \pi\right)$

62. $\sec\theta = \frac{13}{5}, \left(\frac{3\pi}{2}, 2\pi\right)$

Find the exact value of each expression.

63. $\sin 75°$ $\frac{\sqrt{2+\sqrt{3}}}{2}$

64. $\cos\frac{11\pi}{12}$ $-\frac{\sqrt{2+\sqrt{3}}}{2}$

65. $\tan 67.5°$ $\sqrt{2}+1$

66. $\cos\frac{3\pi}{8}$ $\frac{\sqrt{2-\sqrt{2}}}{2}$

67. $\sin\frac{15\pi}{8}$ $-\frac{\sqrt{2-\sqrt{2}}}{2}$

68. $\tan\frac{13\pi}{12}$ $2-\sqrt{3}$

Example 5

Find the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ if θ is in the fourth quadrant and $\tan\theta = -\frac{24}{7}$.

θ is in the fourth quadrant, so $\cos\theta = \frac{7}{25}$ and $\sin\theta = -\frac{24}{25}$.

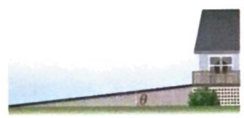
$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\frac{24}{25}\right)\frac{7}{25}$ or $-\frac{336}{625}$

$\cos 2\theta = 2\cos^2\theta - 1$
$= 2\left(\frac{7}{25}\right)^2 - 1$ or $-\frac{527}{625}$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(-\frac{24}{7}\right)}{1-\left(-\frac{24}{7}\right)^2} = \frac{-\frac{48}{7}}{-\frac{527}{49}} \text{ or } \frac{336}{527}$$

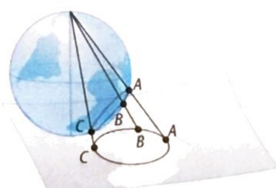
Applications and Problem Solving

69. **CONSTRUCTION** Find the tangent of the angle that the ramp makes with the building if $\sin\theta = \frac{\sqrt{145}}{145}$ and $\cos\theta = \frac{12\sqrt{145}}{145}$. (Lesson 5-1) $\frac{1}{12}$



70. **LIGHT** The intensity of light that emerges from a system of two polarizing lenses can be calculated by $I = I_0 - \frac{I_0}{\csc^2\theta}$, where I_0 is the intensity of light entering the system and θ is the angle of the axis of the second lens with the first lens. Write the equation for the light intensity using only $\tan\theta$. (Lesson 5-1) $I = I_0 - I_0\left(\frac{\tan^2\theta}{1+\tan^2\theta}\right)$

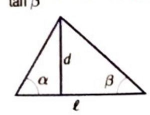
71. **MAP PROJECTIONS** Stereographic projection is used to project the contours of a three-dimensional sphere onto a two-dimensional map. Points on the sphere are related to points on the map using $r = \frac{\sin\alpha}{1-\cos\alpha}$. Verify that $r = \frac{1+\cos\alpha}{\sin\alpha}$. (Lesson 5-2) **See margin.**



72. **PROJECTILE MOTION** A ball thrown with an initial speed v_0 at an angle θ that travels a horizontal distance d will remain in the air t seconds, where $t = \frac{d}{v_0\cos\theta}$. Suppose a ball is thrown with an initial speed of 50 feet per second, travels 100 feet, and is in the air for 4 seconds. Find the angle at which the ball was thrown. (Lesson 5-3) **60°**

73. **BROADCASTING** Interference occurs when two waves pass through the same space at the same time. It is destructive if the amplitude of the sum of the waves is less than the amplitudes of the individual waves. Determine whether the interference is destructive when signals modeled by $y = 20\sin(3t + 45°)$ and $y = 20\sin(3t + 225°)$ are combined. (Lesson 5-4) yes

74. **TRIANGULATION** *Triangulation* is the process of measuring a distance d using the angles α and β and the distance ℓ using $\ell = \frac{d}{\tan\alpha} + \frac{d}{\tan\beta}$. (Lesson 5-5) **See margin.**



a. Solve the formula for d.

b. Verify that $d = \frac{\ell\sin\alpha\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta}$.

c. Verify that $d = \frac{\ell\sin\alpha\sin\beta}{\sin(\alpha+\beta)}$.

d. Show that if $\alpha = \beta$, then $d = 0.5\ell\tan\alpha$.

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5 Practice Test

Find the value of each expression using the given information.

1. $\sin\theta$ and $\cos\theta$, $\csc\theta = -4$, $\cos\theta < 0$ $-\frac{1}{4}, -\frac{\sqrt{15}}{4}$
2. $\csc\theta$ and $\sec\theta$, $\tan\theta = \frac{2}{5}$, $\csc\theta < 0$ $-\frac{\sqrt{29}}{2}, -\frac{\sqrt{29}}{5}$

Simplify each expression.

3. $\frac{\sin(90° - x)}{\tan(90° - x)}$ $\sin x$
4. $\frac{\sec^2 x - 1}{\tan^2 x + 1}$ $\sin^2 x$
5. $\sin\theta(1 + \cot^2\theta)$ $\csc\theta$

Verify each identity. **6–10. See Chapter 5 Answer Appendix.**

6. $\frac{\csc^2\theta - 1}{\csc^2\theta} + \frac{\sec^2\theta - 1}{\sec^2\theta} = 1$
7. $\frac{\cos\theta}{1 + \sin\theta} + \frac{1 - \sin\theta}{\cos\theta} = \frac{2\cos\theta}{1 + \sin\theta}$
8. $\frac{1}{1 + \cos\theta} + \frac{1}{1 - \cos\theta} = 2\csc^2\theta$
9. $-\sec^2\theta\sin^2\theta = \frac{\cos^2\theta - 1}{\cos^2\theta}$
10. $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

11. **MULTIPLE CHOICE** Which expression is *not* true? **D**
    - **A** $\tan(-\theta) = -\tan\theta$
    - **B** $\tan(-\theta) = \frac{1}{\cot(-\theta)}$
    - **C** $\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)}$
    - **D** $\tan(-\theta) + 1 = \sec(-\theta)$

Find all solutions of each equation on the interval $[0, 2\pi]$.

12. $\sqrt{2}\sin\theta + 1 = 0$ $\frac{5\pi}{4}, \frac{7\pi}{4}$
13. $\sec^2\theta = \frac{4}{3}$ $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Solve each equation for all values of θ.

14. $\tan^2\theta - \tan\theta = 0$ $n\pi, \frac{\pi}{4} + n\pi, n \in \mathbb{Z}$
15. $\frac{1 - \sin\theta}{\cos\theta} = \cos\theta$ $n\pi, n \in \mathbb{Z}$
16. $\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} = 2$ $\frac{\pi}{4} + \frac{\pi}{2}n, n \in \mathbb{Z}$
17. $\sec\theta - 2\tan\theta = 0$ $\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, n \in \mathbb{Z}$
18. **CURRENT** The current produced by an alternator is given by $I = 40\sin 135\pi t$, where I is the current in amperes and t is the time in seconds. At what time t does the current first reach 20 amperes? Round to the nearest ten-thousandths. **0.0012 s**

Find the exact value of each trigonometric expression. **19–23. See margin.**

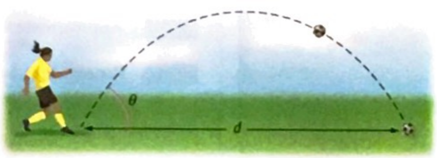
19. $\tan 165°$
20. $\cos -\frac{\pi}{12}$
21. $\sin 75°$
22. $\cos 465° - \cos 15°$
23. $6\sin 675° - 6\sin 45°$

24. **MULTIPLE CHOICE** Which identity is true? **H**
    - **F** $\cos(\theta + \pi) = -\sin\pi$
    - **G** $\cos(\pi - \theta) = \cos\theta$
    - **H** $\sin\left(\theta - \frac{3\pi}{2}\right) = \cos\theta$
    - **J** $\sin(\pi + \theta) = \sin\theta$

Simplify each expression.

25. $\cos\frac{\pi}{8}\cos\frac{3\pi}{8} - \sin\frac{\pi}{8}\sin\frac{3\pi}{8}$ 0
26. $\frac{\tan 135° - \tan 15°}{1 + \tan 135°\tan 15°}$ $-\sqrt{3}$

27. **PHYSICS** A soccer ball is kicked from ground level with an initial speed of v at an angle of elevation θ.



a. The horizontal distance d the ball will travel can be determined using $d = \frac{v^2\sin 2\theta}{g}$, where g is the acceleration due to gravity. Verify that this expression is the same as $\frac{2}{g}v^2(\tan\theta - \tan\theta\sin^2\theta)$. **See margin.**

b. The maximum height h the object will reach can be determined using $h = \frac{v^2\sin^2\theta}{2g}$. Find the ratio of the maximum height attained to the horizontal distance traveled. $\frac{1}{4}\tan\theta$

Find the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ for the given value and interval.

28. $\tan\theta = -3, \left(\frac{3\pi}{2}, 2\pi\right)$ $-\frac{3}{5}, -\frac{4}{5}, \frac{3}{4}$
29. $\cos\theta = \frac{1}{5}, (0°, 90°)$ $\frac{4\sqrt{6}}{25}, -\frac{23}{25}, -\frac{4\sqrt{6}}{23}$
30. $\cos\theta = \frac{5}{9}, \left(0, \frac{\pi}{2}\right)$ $\frac{20\sqrt{14}}{81}; -\frac{31}{81}; -\frac{20\sqrt{14}}{31}$

McGraw-Hill eAssessment Customize and create multiple versions of your chapter test and their answer keys. All of the questions from the leveled chapter tests in the *Chapter 5 Resource Masters* are also available on McGraw-Hill eAssessment.

Additional Answers

19. $-2 + \sqrt{3}$

20. $\frac{\sqrt{6} + \sqrt{2}}{4}$

21. $\frac{\sqrt{6} + \sqrt{2}}{4}$

22. $-\frac{\sqrt{6}}{2}$

23. $-6\sqrt{2}$

27a. $\frac{2}{g}v^2(\tan\theta - \tan\theta\sin^2\theta)$
$= \frac{2}{g}v^2\tan\theta(1 - \sin^2\theta)$
$= \frac{2}{g}v^2\tan\theta\cos^2\theta$
$= \frac{2}{g}v^2\sin\theta\cos\theta$
$= \frac{v^2\sin 2\theta}{g}$

Intervention Planner

TIER 1 On Level OL	TIER 2 Strategic Intervention AL approaching grade level
If students miss about 25% of the exercises or less,	**If** students miss about 50% of the exercises,
Then choose a resource:	**Then** choose a resource:
SE Lessons 5-1, 5-2, 5-3, 5-4, and 5-5 TE Chapter Project, p. 310 connectED.mcgraw-hill.com Self-Check Quiz	Study Guide and Intervention, Chapter 5, pp. 5, 10, 16, 21, and 27 connectED.mcgraw-hill.com Extra Examples, Homework Help