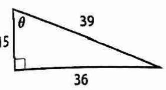
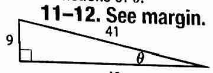
Lesson-by-Lesson Review

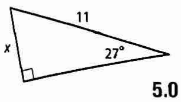
4-1 Right Triangle Trigonometry (pp. 220–230)

Find the exact values of the six trigonometric functions of θ. **11–12. See margin.**

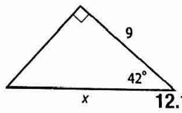
11.


12.


Find the value of x. Round to the nearest tenth, if necessary.

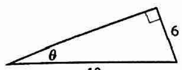
13.


5.0

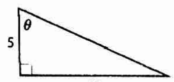
14.


12.1

Find the measure of angle θ. Round to the nearest degree, if necessary.

15.


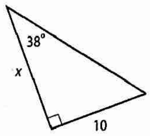
19°

16.


66°

Example 1

Find the value of x. Round to the nearest tenth, if necessary.



$\tan\theta = \frac{\text{opp}}{\text{adj}}$	Tangent function
$\tan 38° = \frac{10}{x}$	$\theta = 38°$, opp $= 10$, and adj $= x$
$x \tan 38° = 10$	Multiply each side by x.
$x = \frac{10}{\tan 38°}$	Divide each side by tan 38°.
$x \approx 12.8$	Use a calculator.

4-2 Degrees and Radians (pp. ...)

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

17. 135° $\frac{3\pi}{4}$

18. 450° $\frac{5\pi}{2}$

19. $\frac{7\pi}{4}$ **315°**

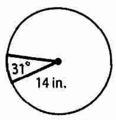
20. $\frac{13\pi}{10}$ 234°

Identify all angles coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle. **21–22. See margin.**

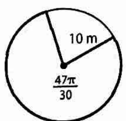
21. 342°

22. $-\frac{\pi}{6}$

Find the area of each sector.

23.


53.0 in²

24.


246.1 m²

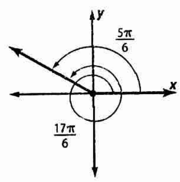
Example

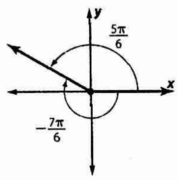
Identify all angles coterminal with $\frac{5\pi}{12}$. Then find and draw one positive and one negative coterminal angle.

All angles measuring $\frac{5\pi}{12} + 2n\pi$ are coterminal with a $\frac{5\pi}{12}$ angle.

Let $n = 1$ and -1.

$\frac{5\pi}{6} + 2\pi(1) = \frac{17\pi}{6}$ $\qquad$ $\frac{5\pi}{6} - 2\pi(-1) = -\frac{7\pi}{6}$





not sufficient to review the topics covered by the questions, remind students that the page references tell them where to review that topic in their textbook.

Two-Day Option Have students complete the Lesson-by-Lesson Review on pp. 303–306. Then you can use McGraw-Hill eAssessment to customize another review worksheet that practices all the objectives of this chapter or only the objectives on which your students need more help.

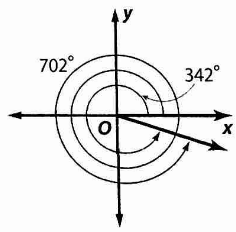
Additional Answers

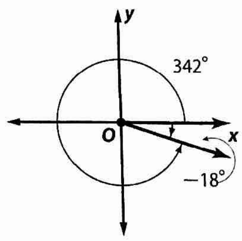
11. $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = \frac{12}{5}$, $\csc\theta = \frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = \frac{5}{12}$

12. $\sin\theta = \frac{9}{41}$, $\cos\theta = \frac{40}{41}$, $\tan\theta = \frac{9}{40}$, $\csc\theta = \frac{41}{9}$, $\sec\theta = \frac{41}{40}$, $\cot\theta = \frac{40}{9}$

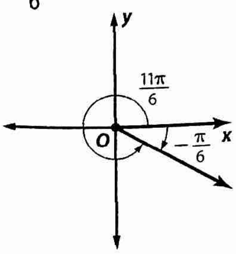
21–22. n is an integer.

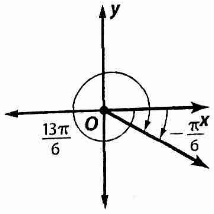
21. $342° + 360n°$; Sample answer: 702°, −18°





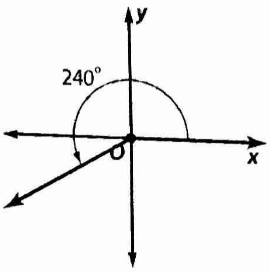
22. $-\frac{\pi}{6} + 2n\pi$; Sample answer: $-\frac{13\pi}{6}, \frac{11\pi}{6}$



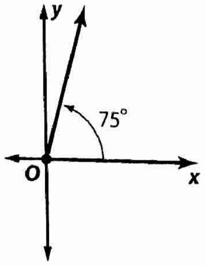


Additional Answers

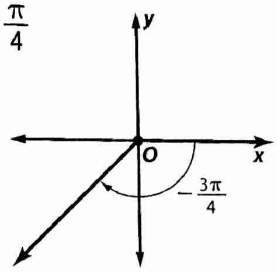
25. 60°



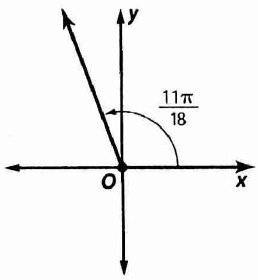
26. 75°



27. $\frac{\pi}{4}$



28. $\frac{7\pi}{18}$



29. $\sin\theta = \frac{\sqrt{21}}{5}$, $\tan\theta = \frac{\sqrt{21}}{2}$, $\csc\theta = \frac{5\sqrt{21}}{21}$, $\sec\theta = \frac{5}{2}$, $\cot\theta = \frac{2\sqrt{21}}{21}$

30. $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

31. $\cos\theta = \frac{12}{13}$, $\csc\theta = -\frac{13}{5}$, $\sec\theta = \frac{13}{12}$, $\tan\theta = -\frac{5}{12}$, $\cot\theta = -\frac{12}{5}$

32. $\sin\theta = -\frac{3\sqrt{13}}{13}$, $\cos\theta = -\frac{2\sqrt{13}}{13}$, $\csc\theta = -\frac{\sqrt{13}}{3}$, $\sec\theta = -\frac{\sqrt{13}}{2}$, $\tan\theta = \frac{3}{2}$

4-3 Trigonometric Functions on the Unit Circle (pp. 242–253)

Sketch each angle. Then find its reference angle.

25. 240°
26. 75° **25–28. See margin.**
27. $-\frac{3\pi}{4}$
28. $\frac{11\pi}{18}$

Find the exact values of the five remaining trigonometric functions of θ. **29–32. See margin.**

29. $\cos\theta = \frac{2}{5}$, where $\sin\theta > 0$ and $\tan\theta > 0$
30. $\tan\theta = -\frac{3}{4}$, where $\sin\theta > 0$ and $\cos\theta < 0$
31. $\sin\theta = -\frac{5}{13}$, where $\cos\theta > 0$ and $\cot\theta < 0$
32. $\cot\theta = \frac{2}{3}$, where $\sin\theta < 0$ and $\tan\theta > 0$

Find the exact value of each expression. If undefined, write *undefined*.

33. $\sin 180°$ **0**
34. $\cot\frac{11\pi}{6}$ $-\sqrt{3}$
35. $\sec 450°$ **undefined**
36. $\cos\left(-\frac{19\pi}{6}\right)$ $-\frac{\sqrt{3}}{2}$

Example 3

Let $\cos\theta = \frac{5}{13}$, where $\sin\theta < 0$. Find the exact values of the five remaining trigonometric functions of θ.

Since $\cos\theta$ is positive and $\sin\theta$ is negative, θ lies in Quadrant IV. This means that the x-coordinate of a point on the terminal side of θ is positive and the y-coordinate is negative.

Since $\cos\theta = \frac{x}{r} = \frac{5}{13}$, use $x = 5$ and $r = 13$ to find y.

$y = \sqrt{r^2 - x^2}$ Pythagorean Theorem

$= \sqrt{169 - 25}$ or 12 $r = 13$ and $x = 5$

$\sin\theta = \frac{y}{r}$ or $\frac{12}{13}$ $\tan\theta = \frac{y}{x}$ or $\frac{12}{5}$ $\sec\theta = \frac{r}{x}$ or $\frac{13}{5}$

$\csc\theta = \frac{r}{y}$ or $\frac{13}{12}$ $\cot\theta = \frac{x}{y}$ or $\frac{5}{12}$

4-4 Graphing Sine and Cosine Functions

Describe how the graphs of $f(x)$ and $g(x)$ are related. Then find the amplitude and period of $g(x)$, and sketch at least one period of both functions on the same coordinate axes. 37–40. See margin.

37. $f(x) = \sin x$, $g(x) = 5\sin x$
38. $f(x) = \cos x$, $g(x) = \cos 2x$
39. $f(x) = \sin x$, $g(x) = \frac{1}{2}\sin x$
40. $f(x) = \cos x$, $g(x) = -\cos\frac{1}{3}x$

State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function.

41. $y = 2\cos(x - \pi)$
42. $y = -\sin 2x + 1$
43. $y = \frac{1}{2}\cos\left(x + \frac{\pi}{2}\right)$
44. $y = 3\sin\left(x + \frac{2\pi}{3}\right)$

41–44. See Chapter 4 Answer Appendix.

Example

State the amplitude, period, frequency, phase shift, and vertical shift of $y = 4\sin\left(x - \frac{\pi}{2}\right) - 4$. Then graph two periods of the function.

In this function, $a = 4$, $b = 1$, $c = -\frac{\pi}{2}$, and $d = -4$.

Amplitude: $|a| = |4|$ or 4

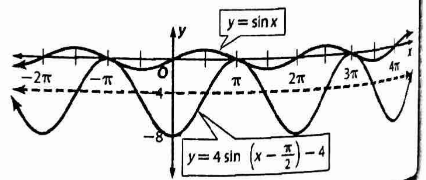
Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|1|}$ or 2π

Frequency: $\frac{|b|}{2\pi} = \frac{|1|}{2\pi}$ or $\frac{1}{2\pi}$

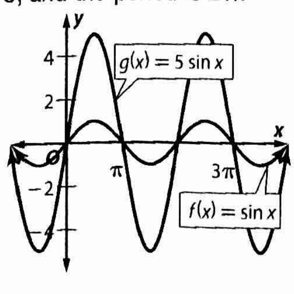
Vertical shift: d or -4

Phase shift: $-\frac{c}{|b|} = -\frac{-\frac{\pi}{2}}{|1|}$ or $\frac{\pi}{2}$

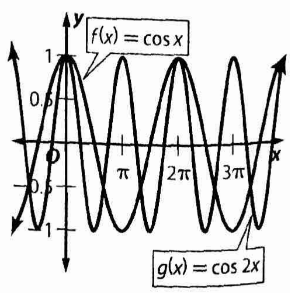
First, graph the midline $y = -4$. Then graph $y = 4\sin x$ shifted $\frac{\pi}{2}$ units to the right and 4 units down.



37. The graph of $g(x)$ is the graph of $f(x)$ expanded vertically. The amplitude of $g(x)$ is 5, and the period is 2π.



38. The graph of $g(x)$ is the graph of $f(x)$ compressed horizontally. The amplitude of $g(x)$ is 1, and the period is π.



4-5 Graphing Other Trigonometric Functions (pp. 269–279)

Locate the vertical asymptotes, and sketch the graph of each function. **45–52. See Chapter 4 Answer Appendix.**

45. $y = 3\tan x$

46. $y = \frac{1}{2}\tan\left(x - \frac{\pi}{2}\right)$

47. $y = \cot\left(x + \frac{\pi}{3}\right)$

48. $y = -\cot(x - \pi)$

49. $y = 2\sec\left(\frac{x}{2}\right)$

50. $y = -\csc(2x)$

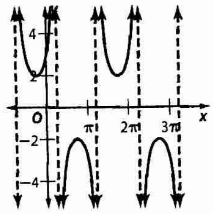
51. $y = \sec(x - \pi)$

52. $y = \frac{2}{3}\csc\left(x + \frac{\pi}{2}\right)$

Example 5

Locate the vertical asymptotes, and sketch the graph of $y = 2\sec\left(x + \frac{\pi}{4}\right)$.

Because the graph of $y = 2\sec\left(x + \frac{\pi}{4}\right)$ is the graph of $y = 2\sec x$ shifted to the left $\frac{\pi}{4}$ units, the vertical asymptotes for one period are located at $-\frac{3\pi}{4}, \frac{\pi}{4}$, and $\frac{5\pi}{4}$. Graph two cycles on the interval $\left[-\frac{3\pi}{4}, \frac{13\pi}{4}\right]$.



4-6 Inverse Trigonometric Functions (pp. 280–290)

Find the exact value of each expression, if it exists.

53. $\sin^{-1}(-1)$ $-\frac{\pi}{2}$

54. $\cos^{-1}\frac{\sqrt{3}}{2}$ $\frac{\pi}{6}$

55. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ $-\frac{\pi}{6}$

56. $\arcsin 0$ 0

57. $\arctan(-1)$ $-\frac{\pi}{4}$

58. $\arccos\frac{\sqrt{2}}{2}$ $\frac{\pi}{4}$

59. $\sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right]$ $-\frac{\pi}{3}$

60. $\cos^{-1}[\cos(-3\pi)]$ π

Example 6

Find the exact value of $\arctan -\sqrt{3}$.

Find a point on the unit circle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with a tangent of $-\sqrt{3}$. When $t = -\frac{\pi}{3}$, $\tan t = -\sqrt{3}$.

Therefore, $\arctan -\sqrt{3} = -\frac{\pi}{3}$.

4-7 The Law of Sines and the Law of Cosines (pp. 291–301)

Find all solutions for the given triangle, if possible. If no solution exists, write *no solution*. Round side lengths to the nearest tenth and angle measurements to the nearest degree. **61–64. See margin.**

61. $a = 11, b = 6, A = 22°$

62. $a = 9, b = 10, A = 42°$

63. $a = 20, b = 10, A = 78°$

64. $a = 2, b = 9, A = 88°$

65–66. See margin.

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

65. $a = 13, b = 12, c = 8$

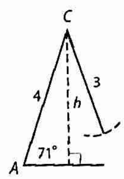
66. $a = 4, b = 5, C = 96°$

Example 7

Solve the triangle if $a = 3$, $b = 4$, and $A = 71°$.

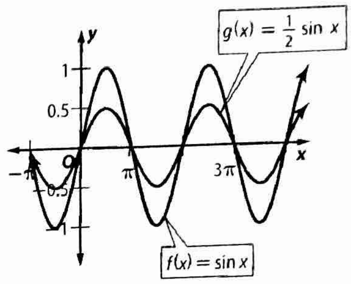
In the figure, $h = 4\sin 71°$ or about 3.8

Because $a \leq h$, there is no triangle that can be formed with sides $a = 3$, $b = 4$, and $A = 71°$. Therefore, this problem has no solution.

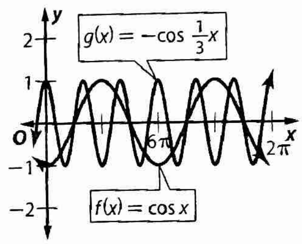


Additional Answers

39. The graph of $g(x)$ is the graph of $f(x)$ compressed vertically. The amplitude of $g(x)$ is $\frac{1}{2}$, and the period is 2π.



40. The graph of $g(x)$ is the graph of $f(x)$ expanded horizontally and reflected in the x-axis. The amplitude of $g(x)$ is 1, and the period is 6π.



61. $B = 12°, C = 146°, c = 16.4$

62. $B = 48°, C = 90°, c = 13.5$ and $B = 132°, C = 6°, c = 1.4$

63. $B = 29°, C = 73°, c = 19.5$

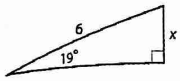
64. no solution

65. $A = 78°, B = 65°, C = 37°$

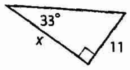
66. $c = 6.7, A = 36°, B = 48°$

Practice Test

Find the value of x. Round to the nearest tenth, if necessary.

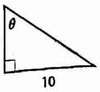
1. 

2.0

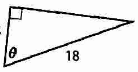
2. 

16.9

Find the measure of angle θ. Round to the nearest degree, if necessary.

3. 

55°

4. 

64°

5. **MULTIPLE CHOICE** What is the linear speed of a point rotating at an angular speed of 36 radians per second at a distance of 12 inches from the center of the rotation? **B**

A 420 in./s

B 432 in./s

C 439 in./s

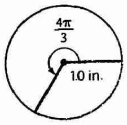
D 444 in./s

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

6. $200°$ $\mathbf{\frac{10\pi}{9}}$

7. $-\frac{8\pi}{3}$ **−480°**

8. Find the area of the sector of the circle shown. 209.4 in^2



Sketch each angle. Then find its reference angle. **9–10. See margin.**

9. $165°$

10. $\frac{21\pi}{13}$

Find the exact value of each expression.

11. $\sec \frac{7\pi}{6}$ $\mathbf{-\frac{2\sqrt{3}}{3}}$

12. $\cos(-240°)$ $-\frac{1}{2}$

13. **MULTIPLE CHOICE** An angle θ satisfies the following inequalities: $\csc \theta < 0$, $\cot \theta > 0$, and $\sec \theta < 0$. In which quadrant does θ lie? **H**

F I

G II

H III

J IV

State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function. **14–15. See margin.**

14. $y = 4 \cos \frac{x}{2} - 5$

15. $y = -\sin\left(x + \frac{\pi}{2}\right)$

16. **TIDES** The table gives the approximate times that the high and low tides occurred in San Azalea Bay over a 2-day period.

Tide	High 1	Low 1	High 2	Low 2
Day 1	2:35 A.M.	8:51 A.M.	3:04 P.M.	9:19 P.M.
Day 2	3:30 A.M.	9:48 A.M.	3:55 P.M.	10:20 P.M.

a. The tides can be modeled with a trigonometric function. Approximately what is the period of this function? **Sample answer: 12 h 30 min**

b. The difference in height between the high and low tides is 7 feet. What is the amplitude of this function? **3.5 ft**

c. Write a function that models the tides where t is measured in hours. Assume the function has no phase shift or vertical shift. $\mathbf{y = 3.5 \sin\left(\frac{4\pi}{25}t\right)}$

Locate the vertical asymptotes, and sketch the graph of each function. **17–18. See margin.**

17. $y = \tan\left(x + \frac{\pi}{4}\right)$

18. $y = \frac{1}{2} \sec 2x$

Find all solutions for the given triangle, if possible. If no solution exists, write *no solution*. Round side lengths to the nearest tenth and angle measurements to the nearest degree. **19. $B = 49°$, $C = 109°$, $c = 20.1$ and $B = 131°$, $C = 27°$, $c = 9.7$**

19. $a = 8, b = 16, A = 22°$

20. $a = 9, b = 7, A = 84°$ **$B = 51°$, $C = 45°$, $c = 6.4$**

21. $a = 3, b = 5, c = 7$ **$A = 22°$, $B = 38°$, $C = 120°$**

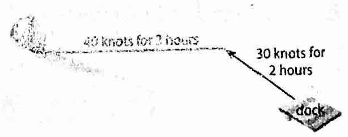
22. $a = 8, b = 10, C = 46°$ **$c = 7.3$, $A = 52°$, $B = 82°$**

Find the exact value of each expression, if it exists.

23. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{5\pi}{6}$

24. $\sin^{-1}\left(-\frac{1}{2}\right)$ $-\frac{\pi}{6}$

25. A boat leaves a dock and travels 45° north of west averaging 30 knots for 2 hours. The boat then travels directly west averaging 40 knots for 3 hours.



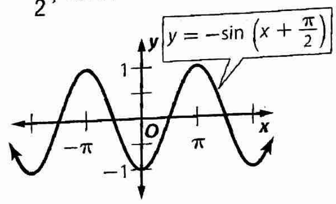
a. How many nautical miles is the boat from the dock after 5 hours? **about 167.9 nautical mi**

b. How many degrees south of east is the dock from the boat's present position? **about 15° south of east**

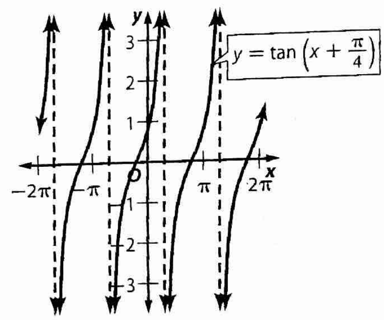
McGraw-Hill eAssessment Customize and create multiple versions of your chapter test and their answer keys. All of the questions from the leveled chapter tests in the *Chapter 4 Resource Masters* are also available on McGraw-Hill eAssessment.

Additional Answers

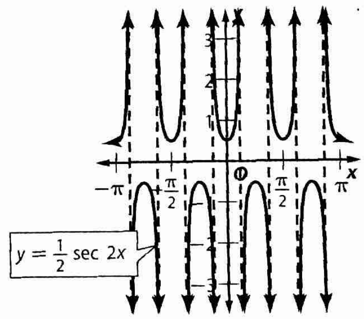
15. amplitude $= 1$; period $= 2\pi$; frequency $= \frac{1}{2\pi}$; phase shift $= -\frac{\pi}{2}$; vertical shift $= 0$;



17.



18.



Intervention Planner

TIER 1 On Level OL

If students miss about 25% of the exercises or less,

Then choose a resource:

SE Lessons 4-1, 4-2, 4-3, 4-4, 4-5, 4-6 and 4-7

Practice, Chapter 4, pp. 7, 12, 17, 22, 28, 34, and 39

TE Chapter Project, p. 218

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TIER 2 Strategic Intervention AL approaching grade level

If students miss about 50% of the exercises,

Then choose a resource:

Study Guide and Intervention, Chapter 4, pp. 5, 10, 15, 20, 26, 32, and 37

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