

Locate the vertical asymptotes, and sketch the graph of each function. **45–52. See Chapter 4 Answer Appendix.**

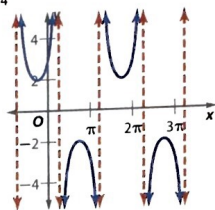
45.  $y = 3 \tan x$       46.  $y = \frac{1}{2} \tan \left(x - \frac{\pi}{2}\right)$   
 47.  $y = \cot \left(x + \frac{\pi}{3}\right)$       48.  $y = -\cot \left(x - \pi\right)$   
 49.  $y = 2 \sec \left(\frac{x}{2}\right)$       50.  $y = -\csc(2x)$   
 51.  $y = \sec(x - \pi)$       52.  $y = \frac{2}{3} \csc \left(x + \frac{\pi}{2}\right)$

**Example 5**

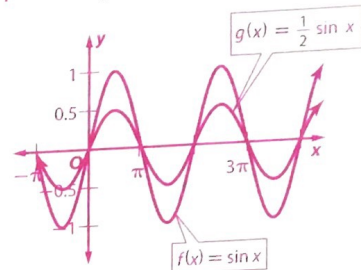
Locate the vertical asymptotes, and sketch the graph of  $y = 2 \sec \left(x + \frac{\pi}{4}\right)$ .

Because the graph of  $y = 2 \sec \left(x + \frac{\pi}{4}\right)$  is the graph of  $y = 2 \sec x$  shifted to the left  $\frac{\pi}{4}$  units, the vertical asymptotes for one period are located at  $-\frac{3\pi}{4}, \frac{\pi}{4}$ , and  $\frac{5\pi}{4}$ .

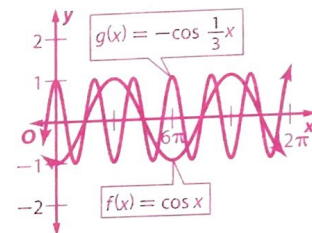
Graph two cycles on the interval  $\left[-\frac{3\pi}{4}, \frac{13\pi}{4}\right]$ .



amplitude of  $g(x)$  is  $\frac{1}{2}$ , and the period is  $2\pi$ .



40. The graph of  $g(x)$  is the graph of  $f(x)$  expanded horizontally and reflected in the  $x$ -axis. The amplitude of  $g(x)$  is 1, and the period is  $6\pi$ .



61.  $B = 12^\circ, C = 146^\circ, c = 16.4$   
 62.  $B = 48^\circ, C = 90^\circ, c = 13.5$  and  $B = 132^\circ, C = 6^\circ, c = 1.4$   
 63.  $B = 29^\circ, C = 73^\circ, c = 19.5$   
 64. no solution  
 65.  $A = 78^\circ, B = 65^\circ, C = 37^\circ$   
 66.  $c = 6.7, A = 36^\circ, B = 48^\circ$

**4-6 Inverse Trigonometric Functions** (pp. 280–290)

Find the exact value of each expression, if it exists.

53.  $\sin^{-1}(-1) = -\frac{\pi}{2}$       54.  $\cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$   
 55.  $\tan^{-1} \left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$       56.  $\arcsin 0 = 0$   
 57.  $\arctan(-1) = -\frac{\pi}{4}$       58.  $\arccos \frac{\sqrt{2}}{2} = \frac{\pi}{4}$   
 59.  $\sin^{-1} \left[\sin \left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$       60.  $\cos^{-1} [\cos(-3\pi)] = \pi$

**Example 6**

Find the exact value of  $\arctan -\sqrt{3}$ .

Find a point on the unit circle in the interval  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  with a tangent of  $-\sqrt{3}$ . When  $t = -\frac{\pi}{3}$ ,  $\tan t = -\sqrt{3}$ .

Therefore,  $\arctan -\sqrt{3} = -\frac{\pi}{3}$ .

**4-7 The Law of Sines and the Law of Cosines** (pp. 291–301)

Find all solutions for the given triangle, if possible. If no solution exists, write *no solution*. Round side lengths to the nearest tenth and angle measurements to the nearest degree. **61–64. See margin.**

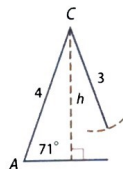
61.  $a = 11, b = 6, A = 22^\circ$       62.  $a = 9, b = 10, A = 42^\circ$   
 63.  $a = 20, b = 10, A = 78^\circ$       64.  $a = 2, b = 9, A = 88^\circ$   
**65–66. See margin.**  
 Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.  
 65.  $a = 13, b = 12, c = 8$       66.  $a = 4, b = 5, C = 96^\circ$

**Example 7**

Solve the triangle if  $a = 3, b = 4$ , and  $A = 71^\circ$ .

In the figure,  $h = 4 \sin 71^\circ$  or about 3.8

Because  $a \leq h$ , there is no triangle that can be formed with sides  $a = 3, b = 4$ , and  $A = 71^\circ$ . Therefore, this problem has no solution.



## Lesson-by-Lesson Review

**Intervention** If the given examples are not sufficient to review the topics covered by the questions, remind students that the page references tell them where to review that topic in their textbooks.

**Two-Day Option** Have students complete the Lesson-by-Lesson Review on pp. 526–528. Then you can use McGraw-Hill eAssessment to customize another review worksheet that practices all the objectives of this chapter or only the objectives on which your students need more help.

### Additional Answers

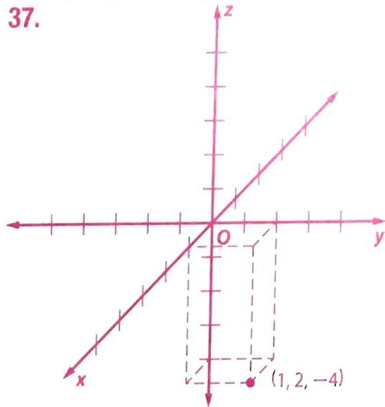
27.  $\left\langle \frac{7\sqrt{53}}{53}, \frac{2\sqrt{53}}{53} \right\rangle$

28.  $\left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$

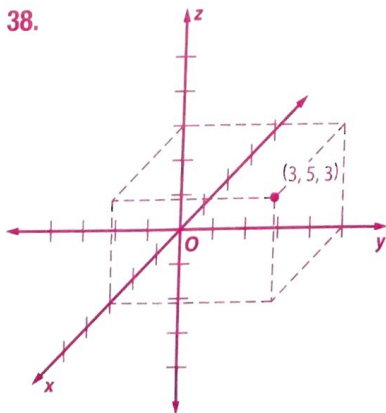
29.  $\left\langle \frac{5\sqrt{89}}{89}, -\frac{8\sqrt{89}}{89} \right\rangle$

30.  $\left\langle \frac{3\sqrt{10}}{10}, \frac{\sqrt{10}}{10} \right\rangle$

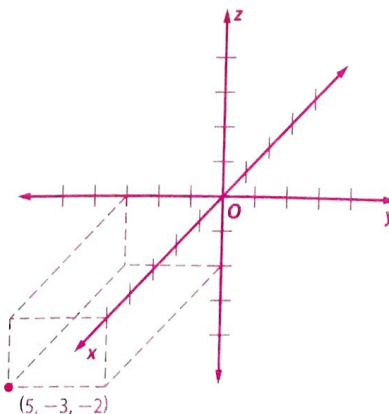
37.



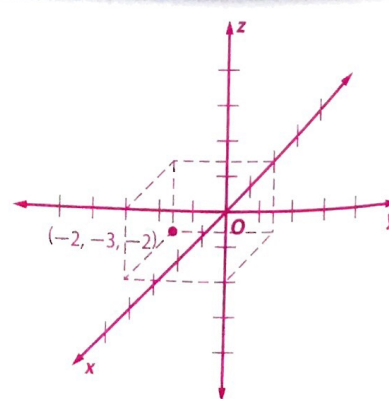
38.



39.



40.



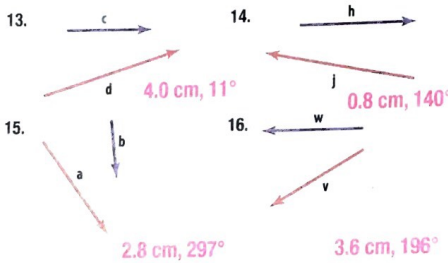
## Lesson-by-Lesson Review

### 8-1 Introduction to Vectors (pp. 482–491)

State whether each quantity described is a **vector** quantity or a **scalar** quantity.

- a car driving 50 miles an hour due east **vector**
- a gust of wind blowing 5 meters per second **scalar**

Find the resultant of each pair of vectors using either the triangle or parallelogram method. State the magnitude of the resultant to the nearest tenth of a centimeter and its direction relative to the horizontal.

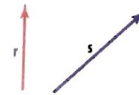


Determine the magnitude and direction of the resultant of each vector sum.

- 70 meters due west and then 150 meters due east **80 m due east**
- 8 newtons directly backward and then 12 newtons directly backward **20 N backward**

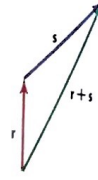
### Example 1

Find the resultant of  $r$  and  $s$  using either the triangle or parallelogram method. State the magnitude of the resultant in centimeters and its direction relative to the horizontal.



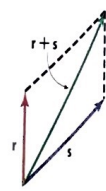
#### Triangle Method

Translate  $s$  so that the tip of  $r$  touches the tail of  $s$ . The resultant is the vector from the tail of  $r$  to the tip of  $s$ .



#### Parallelogram Method

Translate  $s$  so that the tail of  $s$  touches the tail of  $r$ . Complete the parallelogram that has  $r$  and  $s$  as two of its sides. The resultant is the vector that forms the indicated diagonal of the parallelogram.



The magnitude of the resultant is 3.4 cm and the direction is  $59^\circ$ .

19.  $\langle 6, 1 \rangle$ ;  $\sqrt{37} \approx 6.1$     20.  $\langle -16, 8 \rangle$ ;  $8\sqrt{5} \approx 17.9$

### Vectors in the Coordinate Plane (pp. 492–499)

Find the component form and magnitude of  $\overline{AB}$  with the given initial and terminal points.

- $A(-1, 3), B(5, 4)$
  - $A(7, -2), B(-9, 6)$
  - $A(-8, -4), B(6, 1)$
  - $A(2, -10), B(3, -5)$
- $\langle 14, 5 \rangle$ ;  $\sqrt{221} \approx 14.9$      $\langle 1, 5 \rangle$ ;  $\sqrt{26} \approx 5.1$

Find each of the following for  $p = \langle 4, 0 \rangle$ ,  $q = \langle -2, -3 \rangle$ , and  $t = \langle -4, 2 \rangle$ .

- $2q - p$   $\langle -8, -6 \rangle$
- $p + 2t$   $\langle -4, 4 \rangle$
- $t - 3p + q$   $\langle -18, -1 \rangle$
- $2p + t - 3q$   $\langle 10, 11 \rangle$

Find a unit vector  $u$  with the same direction as  $v$ .

- $v = \langle -7, 2 \rangle$
  - $v = \langle 3, -3 \rangle$
  - $v = \langle -5, -8 \rangle$
  - $v = \langle 9, 3 \rangle$
- 27–30. See margin.

### Example 2

Find the component form and magnitude of  $\overline{AB}$  with initial point  $A(3, -2)$  and terminal point  $B(4, -1)$ .

$$\begin{aligned} \overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle && \text{Component form} \\ &= \langle 4 - 3, -1 - (-2) \rangle && \text{Substitute.} \\ &= \langle 1, 1 \rangle && \text{Simplify.} \end{aligned}$$

Find the magnitude using the Distance Formula.

$$\begin{aligned} |\overline{AB}| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{[(4 - 3)]^2 + [-1 - (-2)]^2} && \text{Substitute.} \\ &= \sqrt{2} \text{ or about } 1.4 && \text{Simplify.} \end{aligned}$$

### 8-3 Dot Products and Vector Projections (pp. 500–508)

Find the dot product of  $\mathbf{u}$  and  $\mathbf{v}$ . Then determine if  $\mathbf{u}$  and  $\mathbf{v}$  are orthogonal.

31.  $\mathbf{u} = \langle -3, 5 \rangle, \mathbf{v} = \langle 2, 1 \rangle$     **-1; not orthogonal**  
 32.  $\mathbf{u} = \langle 4, 4 \rangle, \mathbf{v} = \langle 5, 7 \rangle$     **48; not orthogonal**  
 33.  $\mathbf{u} = \langle -1, 4 \rangle, \mathbf{v} = \langle 8, 2 \rangle$     **0; orthogonal**  
 34.  $\mathbf{u} = \langle -2, 3 \rangle, \mathbf{v} = \langle 1, 3 \rangle$     **7; not orthogonal**

Find the angle  $\theta$  between  $\mathbf{u}$  and  $\mathbf{v}$  to the nearest tenth of a degree.

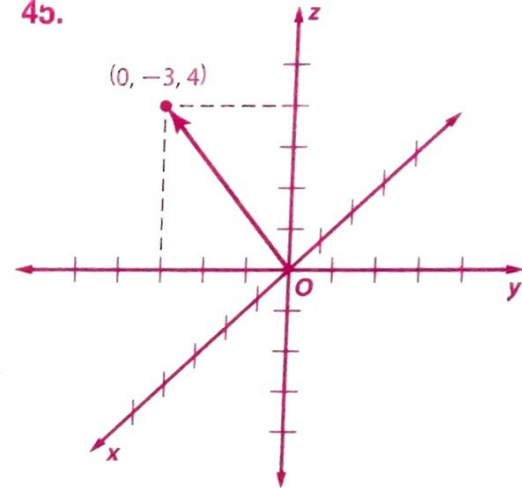
35.  $\mathbf{u} = \langle 5, -1 \rangle, \mathbf{v} = \langle -2, 3 \rangle$     **135.0°**  
 36.  $\mathbf{u} = \langle -1, 8 \rangle, \mathbf{v} = \langle 4, 2 \rangle$     **70.6°**

#### Example 3

Find the dot product of  $\mathbf{x} = \langle 2, -5 \rangle$  and  $\mathbf{y} = \langle -4, 7 \rangle$ . Then determine if  $\mathbf{x}$  and  $\mathbf{y}$  are orthogonal.

$$\begin{aligned} \mathbf{x} \cdot \mathbf{y} &= x_1 y_1 + x_2 y_2 && \text{Dot product} \\ &= 2(-4) + -5(7) && \text{Substitute.} \\ &= -8 + (-35) \text{ or } -43 && \text{Simplify.} \end{aligned}$$

Since  $\mathbf{x} \cdot \mathbf{y} \neq 0$ ,  $\mathbf{x}$  and  $\mathbf{y}$  are not orthogonal.



### 8-4 Vectors in Three-Dimensional Space (pp. 510–516)

Plot each point in a three-dimensional coordinate system.

37.  $(1, 2, -4)$     38.  $(3, 5, 3)$     **37–40. See margin.**  
 39.  $(5, -3, -2)$     40.  $(-2, -3, -2)$

#### 41. $2\sqrt{38} \approx 12.3$ ; $(-1, 5, 6)$

Find the length and midpoint of the segment with the given endpoints.

41.  $(-4, 10, 4), (2, 0, 8)$      **$2\sqrt{29} \approx 10.8$ ;  $(-7, 2, 1)$**   
 42.  $(-5, 6, 4), (-9, -2, -2)$   
 43.  $(3, 2, 0), (-9, -10, 4)$      **$4\sqrt{19} \approx 17.4$ ;  $(-3, -4, 2)$**   
 44.  $(8, 3, 2), (-4, -6, 6)$      **$\sqrt{241} \approx 15.5$ ;  $(2, -1.5, 4)$**

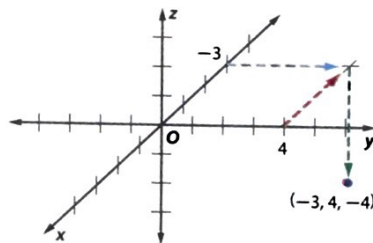
Locate and graph each vector in space. **45–48. See margin.**

45.  $\mathbf{a} = \langle 0, -3, 4 \rangle$     46.  $\mathbf{b} = -3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$   
 47.  $\mathbf{c} = -2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$     48.  $\mathbf{d} = \langle -4, -5, -3 \rangle$

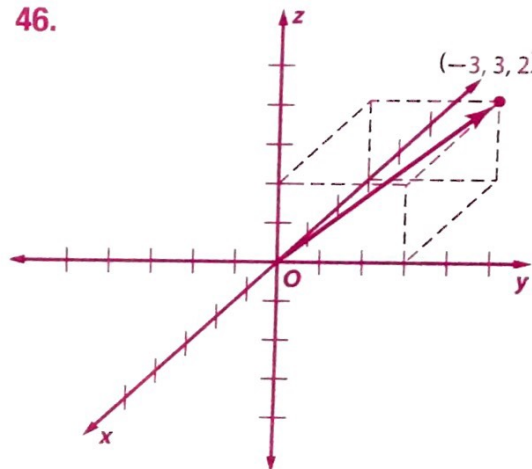
#### Example 4

Plot  $(-3, 4, -4)$  in a three-dimensional coordinate system.

Locate the point  $(-3, 4)$  in the  $xy$ -plane and mark it with a cross. Then plot a point 4 units down from this location parallel to the  $z$ -axis.



46.



### 8-5 Vectors in Three-Dimensional Space (pp. 518–524)

Find the dot product of  $\mathbf{u}$  and  $\mathbf{v}$ . Then determine if  $\mathbf{u}$  and  $\mathbf{v}$  are orthogonal.

49.  $\mathbf{u} = \langle 2, 5, 2 \rangle, \mathbf{v} = \langle 8, 2, -13 \rangle$     **0; orthogonal**  
 50.  $\mathbf{u} = \langle 5, 0, -6 \rangle, \mathbf{v} = \langle -6, 1, 3 \rangle$     **-48; not orthogonal**

Find the cross product of  $\mathbf{u}$  and  $\mathbf{v}$ . Then show that  $\mathbf{u} \times \mathbf{v}$  is orthogonal to both  $\mathbf{u}$  and  $\mathbf{v}$ . **51–52. See margin.**

51.  $\mathbf{u} = \langle 1, -3, -2 \rangle, \mathbf{v} = \langle 2, 4, -3 \rangle$   
 52.  $\mathbf{u} = \langle 4, 1, -2 \rangle, \mathbf{v} = \langle 5, -4, -1 \rangle$

#### Example 5

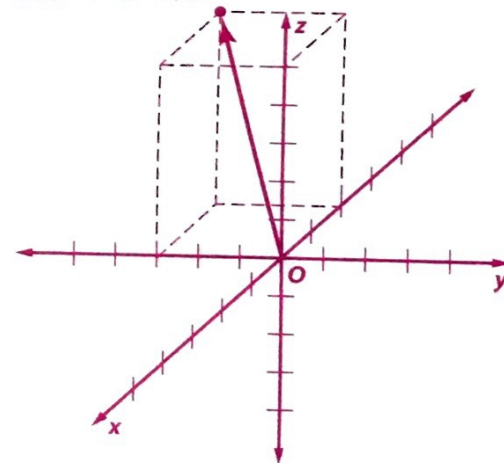
Find the cross product of  $\mathbf{u} = \langle -4, 2, -3 \rangle$  and  $\mathbf{v} = \langle 7, 11, 2 \rangle$ . Then show that  $\mathbf{u} \times \mathbf{v}$  is orthogonal to both  $\mathbf{u}$  and  $\mathbf{v}$ .

$$\begin{aligned} \mathbf{u} \times \mathbf{v} &= \begin{vmatrix} 2 & -3 \\ 11 & 2 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -4 & -3 \\ 7 & 2 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -4 & 2 \\ 7 & 11 \end{vmatrix} \mathbf{k} \\ &= \langle 37, -13, -58 \rangle \end{aligned}$$

$$\begin{aligned} (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} &= \langle 37, -13, -58 \rangle \cdot \langle -4, 2, -3 \rangle \\ &= -148 - 26 + 174 \text{ or } 0 \checkmark \end{aligned}$$

$$\begin{aligned} (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} &= \langle 37, -13, -58 \rangle \cdot \langle 7, 11, 2 \rangle \\ &= 259 - 143 - 116 \text{ or } 0 \checkmark \end{aligned}$$

47.  $(-2, -3, 5)$



48.

