Thursday, September 21, 2017 11:45 AM

# 2.6 Notes: Writing Polynomial Equations

**Conjugate Root Theorem:** Suppose $P(x)$ is a polynomial w/ real coefficients, if $P(a+ib)=0$ and/or $P(a+b\sqrt{c})=0$, then $P(a-ib)=0$ and/or $P(a-b\sqrt{c})=0$

* Imaginary and irrational roots/zeros come in pairs!!

ex:
If $-i$ is a solution, then $i$ is a solution
If $2-\sqrt{3}$ is a solution, then $2+\sqrt{3}$ is a solution

To write polynomial equations given roots/zeros:
1. Set each root/zero $= x$
2. "work backwards" and move all #s to the same side as $x$ (should get 0 on the right side)
3. Box or foil factors together
   - Be sure to multiply irrational w/ irrational or imaginary w/ imaginary, FIRST

Recall...

$\sqrt{-1} = i$ $\qquad$ $2\sqrt{3}-5\sqrt{3}=-3\sqrt{3}$

$-1 = i^2$ $\qquad$ $3\sqrt{5}+6\sqrt{3}$

Find each polynomail given the roots:

1. $x = 5, -2, \frac{3}{4}$

$x=5$ $\qquad$ $x=-2$ $\qquad$ $4(x)=\left(\frac{3}{4}\right)4$

$-5\ -5$ $\qquad$ $+2\ +2$ $\qquad$ $4x=3$

$-3\ -3$

$x-5=0$ $\qquad$ $x+2=0$ $\qquad$ $4x-3=0$

$x \quad -5$

$x \mid x^2 \mid -5x$ $\qquad$ $x^2-3x-10$

| | $x$ | $-5$ |
|---|---|---|
| $x$ | $x^2$ | $-5x$ |
| $2$ | $2x$ | $-10$ |

$x^2 - 3x - 10$

| | $x^2$ | $-3x$ | $-10$ |
|---|---|---|---|
| $4x$ | $4x^3$ | $-12x^2$ | $-40x$ |
| $-3$ | $-3x^2$ | $9x$ | $30$ |

$$\boxed{4x^3 - 15x^2 - 31x + 30}$$

② $x = -\frac{1}{5}, 0, 1, \frac{2}{3}$

$5(x) = \left(-\frac{1}{5}\right)5$ $x = 0$ $x = 1$ $3(x) = \left(\frac{2}{3}\right)3$

$5x = -1$ $3x = 2$

$5x + 1 = 0$ $x = 0$ $x - 1 = 0$ $3x - 2 = 0$

| | $x$ | $-1$ |
|---|---|---|
| $3x$ | $3x^2$ | $-3x$ |
| $-2$ | $-2x$ | $2$ |

$3x^2 - 5x + 2$

| | $15x^3$ | $-25x^2$ | $10x$ |
|---|---|---|---|
| $5x$ | | | |
| $1$ | $3x^2$ | $-5x$ | $2$ |

$$x(15x^3 - 22x^2 + 5x + 2) = \boxed{15x^4 - 22x^3 + 5x^2 + 2x}$$

③ $x = 2 + \sqrt{13}, -3$

$2 - \sqrt{13}$

$x = 2 + \sqrt{13}$ $x = 2 - \sqrt{13}$ $x = -3$

$x - 2 - \sqrt{13} = 0$ $x - 2 + \sqrt{13} = 0$ $x + 3 = 0$

| | $x$ | $-2$ | $-\sqrt{13}$ |
|---|---|---|---|
| $x$ | $x^2$ | $-2x$ | $-x\sqrt{13}$ |
| $-2$ | $-2x$ | $4$ | $2\sqrt{13}$ |

$x^2 - 4x - 9$

| | $x^3$ | $-4x^2$ | $-9x$ |
|---|---|---|---|
| $x$ | | | |
| $2$ | | | |

| | | | |
|---|---|---|---|
| $-2$ | $-2x$ | $4$ | $2\sqrt{13}$ |
| $\sqrt{13}$ | $x\sqrt{13}$ | $2\sqrt{13}$ | $-\sqrt{169}$ $-13$ |

| | | | |
|---|---|---|---|
| $x$ | $-7x$ | $-7x$ | |
| $3$ | $3x^2$ | $-12x$ | $-27$ |

$$x^3 - x^2 - 21x - 27$$

④ $x = -\frac{3}{2}, 1 - 3\sqrt{2}, 4$
$1 + 3\sqrt{2}$

$x = -\frac{3}{2}$ $\quad x = 1 - 3\sqrt{2}$ $\quad x = 1 + 3\sqrt{2}$ $\quad x = 4$

$2x + 3 = 0$ $\quad x - 1 + 3\sqrt{2} = 0$ $\quad x - 1 - 3\sqrt{2} = 0$ $\quad x - 4 = 0$

| | $x$ | $-1$ | $3\sqrt{2}$ |
|---|---|---|---|
| $x$ | $x^2$ | $-x$ | $3x\sqrt{2}$ |
| $-1$ | $-x$ | $1$ | $-3\sqrt{2}$ |
| $-3\sqrt{2}$ | $-3x\sqrt{2}$ | $3\sqrt{2}$ | $-18$ |

$-9\sqrt{4}$

$$x^2 - 2x - 17$$

| | $2x$ | $3$ |
|---|---|---|
| $x$ | $2x^2$ | $3x$ |
| $-4$ | $-8x$ | $-12$ |

$$2x^2 - 5x - 12$$

| | $2x^2$ | $-5x$ | $-12$ |
|---|---|---|---|
| $x^2$ | $2x^4$ | $-5x^3$ | $-12x^2$ |
| $-2x$ | $-4x^3$ | $10x^2$ | $24x$ |
| $-17$ | $-34x^2$ | $85x$ | $204$ |

$$2x^4 - 9x^3 - 36x^2 + 109x + 204$$

⑤ $x = -11, -5i$

$+5i$

$x = -11$ $\quad x = -5i$ $\quad x = 5i$

$x + 11 = 0$ $\quad x + 5i = 0$ $\quad x - 5i = 0$

| | $x$ | $5i$ |
|---|---|---|
| $x$ | $x^2$ | ~~$5xi$~~ |
| $-5i$ | ~~$-5xi$~~ | $25$ |

$-25i^2$
$-25(-1)$

$x^2 + 25$

| | $x^3$ | $25x$ |
|---|---|---|
| $x$ | | |
| $11$ | $11x^2$ | $275$ |

$x^3 + 11x^2 + 25x + 275$

⑥ $m = 0, 7, 3+5i$

$m = 0$ $\quad m = 7$ $\quad m = 3+5i$ $\quad m = 3-5i$

$m = 0$ $\quad m - 7 = 0$ $\quad m - 3 - 5i = 0$ $\quad m - 3 + 5i = 0$

| | $m^2$ | $-6m$ | $34$ |
|---|---|---|---|
| $m$ | $m^3$ | ~~$-6m^2$~~ | ~~$34m$~~ |
| $-7$ | ~~$-7m^2$~~ | $42m$ | $-238$ |

$m(m^3 - 13m^2 + 76m - 238)$

$m^4 - 13m^3 + 76m^2 - 238m$

| | $m$ | $-3$ | $-5i$ |
|---|---|---|---|
| $m$ | $m^2$ | $-3m$ | ~~$-5mi$~~ |
| $-3$ | $-3m$ | $9$ | ~~$15i$~~ |
| $5i$ | ~~$5mi$~~ | ~~$-15i$~~ | $25$ |

$-25(i^2)$
$-25(-1)$

$m^2 - 6m + 34$

$m^2 - 6m + 34$

$m^4 - 13m^3 + 76m^2 - 238m$

⑦ $K = -\frac{7}{3}, \; -5 + 2i\sqrt{3}, \; -5 - 2i\sqrt{3}$

$3(K) = \left(-\frac{7}{3}\right)3$ $\quad K = -5 + 2i\sqrt{3}$ $\quad K = -5 - 2i\sqrt{3}$

$3K = -7$

$3K + 7 = 0$ $\quad K + 5 - 2i\sqrt{3} = 0$ $\quad K + 5 + 2i\sqrt{3} = 0$

| | $K^2$ | $10K$ | $37$ |
|---|---|---|---|
| $3K$ | $3K^3$ | $30K^2$ | $111K$ |
| $7$ | $7K^2$ | $70K$ | $259$ |

| | $K$ | $5$ | $-2i\sqrt{3}$ |
|---|---|---|---|
| $K$ | $K^2$ | $5K$ | $-2Ki\sqrt{3}$ |
| $5$ | $5K$ | $25$ | $-10i\sqrt{3}$ |
| $2i\sqrt{3}$ | $2Ki\sqrt{3}$ | $10i\sqrt{3}$ | $12$ |

$-4i^2\sqrt{9}$

$-4(-1)(3)$

$K^2 + 10K + 37$

$3K^3 + 37K^2 + 181K + 259$